ABSTRACTS

(*in alphabetical order*)

Mathematical Modeling of Vaccine Noncompliance

Jordan Bauer*, *Valparaiso University*

Vaccine scares can prevent individuals from complying with a vaccination program. When compliance is high, the critical vaccination proportion is close to being met, and herd immunity occurs, bringing the disease incidence to extremely low levels. Thus, the risk to vaccinate may seem greater than the risk of contracting the disease, inciting vaccine noncompliance. A previous behavior-incidence ordinary differential equation model shows both social learning and feedback contributing to changes in vaccinating behavior, where social learning is the perceived risk of vaccinating and feedback represents new cases of the disease. In our study, we compared several candidate models to more simply illustrate both vaccination coverage and incidence through social learning and feedback. The behavior model uses logistic growth and exponential decay to describe the social learning aspect as well as different functional forms of the disease prevalence to represent feedback. Each candidate model was tested by fitting it to data from the pertussis vaccine scare in England and Wales in the 1970s. Our most parsimonious model shows a superior fit to the vaccine coverage curve during the scare.

*Research Institution: Valparaiso University
Research Advisor: Alex Capaldi

Upper and lower bounds on the speed of a one dimensional excited random walk

Erin Bossen*, *Eastern Illinois University*
Brian Kidd*, *Purdue University*
Owen Levin*, *University of Minnesota*
Jacob Smith*, *Franklin College*
Kevin Stangl*, *UCLA*

Random walks are used to model diverse phenomena across many fields including physics, chemistry, ecology, and mathematical finance. The classic random walk is a Markovian stochastic process, where the probability of moving forward or backward is homogenous in time. We are exploring excited random walks (also called cookie random walks), a non-Markovian stochastic process that models motion where past behavior of the walk influences future behavior in that the number of times the walker visited a site determines the transition probability of the next step of the walker. Certain key properties of the walk, such as the recurrence and transience have been explicitly determined as a function of the model parameters. There exists a probabilistic formula for the speed of the walker, but this formula has not been explicitly computed. The main thrust of our research effort is focused on obtaining tight bounds for the speed of the random walker using branching processes and exploring qualitative properties of the process, including differentiability, concavity, and monotonicity of the speed function.

*Research Institution: Purdue University
Research Advisor: Jonathon Peterson

Geometry of Hyperbolic Percolation Clusters
Emma Brissett [*], *Benedictine College*

Given an infinite connected graph G, we can perform a Bernoulli bond percolation on G in the following way: for each edge in G, with probability p, the edge remains, and with probability p-1, the edge is removed. We are left with a random subgraph of G containing a combination of finite and infinite components called clusters. Every infinite connected graph has constants p_c and p_u in [0, 1] which are threshold probability values with respect to the number of infinite clusters in the percolation subgraph. Of special interest are percolations on hyperbolic graphs. There are many results concerning the number of infinite clusters in percolation subgraphs of planar hyperbolic graphs, as well as p_c and p_u values for such graphs, and we extend these to include nonplanar hyperbolic graphs. From there, we study the asymptotic behavior of infinite clusters in Gromov-hyperbolic graphs and homogeneous graphs with exponential growth.

[*]Research Institution: Indiana University
Research Advisor: Chris Connell

Theoretical Model of Flow Compensation following Vascular Occlusion
Myson Burch*, *IUPUI*
Elizabeth Franko*, *University of Scranton*

Peripheral arterial disease (PAD) is a major health problem in which arteries within the systemic vasculature become partially or fully blocked, often due to atherosclerosis, leading to a significant reduction in blood flow to tissue. Patients often require surgical bypass grafts to restore flow to their tissue; in extreme cases, amputation is required. The absence of data regarding the relative importance of adaptations in collateral arteries, arterioles, and capillaries to compensation after arterial occlusion is a major roadblock for the development of successful and noninvasive therapies for PAD patients. The objective of this project is to integrate experimental and theoretical techniques to assess the significance of changes in vascular segments at rest and during exercise subsequent to a major arterial occlusion on an acute and chronic time frame. Model-predicted values of vascular resistance are compared with experimental studies to validate the model. The model is extended to predict changes in vessel diameter according to mechanistic responses to pressure, shear stress, and metabolism following an occlusion. Theoretical results suggest that therapies that increase collateral diameter in combination with distal microcirculation adaptations provide the maximum benefit to patients with PAD. Ultimately this project offers a first step in optimizing experimental design and diagnostic criteria to focus on the most relevant vascular segments in studies of vascular compensation in health and disease.

[*]Research Institution: IUPUI
Research Advisor: Julia Arciero

Searching for Solitary Pseudo-Anosovs
Yvonne Chazal[*], *North Carolina State University*

Of the three types of self-homeomorphisms, pseudo-Anosov is both the most prevalent and the most mysterious. Given a random self-homeomorphism, it is most likely pseudo-Anosov, but proving it is such can be quite difficult. These fascinating functions can be represented by their action on certain polygons. In many cases, pseudo-Anosovs share the same polygonal representation. If a pseudo-Anosov does not share its polygonal representation with any other pseudo-Anosov, we will call it solitary. No solitary pseudo-Anosovs have yet been found, but we have found some clues as to where they might be hiding.

[*]Research Institution: Indiana University
Research Advisor: Chris Judge

Nucleolus and Group Monotonicity
Matthew Chen*, *Goshen College*

A coalition game (N, v) models a situation in which gains can be made by collaboration: $N = \{1, 2, \ldots, n\}$ is the set of players and $v(S)$ is the real-number gain achievable for each nonempty subset S of players (called coalitions). We assume that $v(S) + v(T) \leq v(S \cup T)$ for each pair of disjoint coalitions S and T because one way for the subset $S \cup T$ of players to collaborate is to work in the separate groups S and T. With this assumption, all players collaborating results in the largest combined payoff $v(N)$. Now the question is how to fairly allocate the gains obtained through collaboration, that is, what payoffs $x_1, x_2, \ldots, x_n$ should be given to the players subject to the constraint $\sum_{i \in N} x_i = v(N)$? The nucleolus $\nu(N, v)$ is an allocation method that is group rational: $\sum_{i \in S} \nu_i(N, v) \geq v(S)$ for all coalitions S whenever such an allocation exists. The excess of some allocation x on some coalition S, or $e(x,S)$, is defined as $v(S) - \sum_{i \in N} x_i$, which specifies the "dissatisfaction" that the players in S have against x. The nucleolus is the allocation that successively minimizes the largest excesses. Unfortunately, the nucleolus is not always group monotone: an increase in one of the coalition gains $v(S)$ may result in a decrease in the nucleolus allocation $\nu_i(N, v)$ for some player $i \in S$. We try to characterize the 4-player coalition games at which the nucleolus is group monotone.

*Research Institution: Goshen College
Research Advisor: David Housman

Modeling Marine Bacteria using Chemotaxis
Margaret Christy*, *Purdue University*
Gregory Javens*, *Hunter College*

Chemotactic bacteria rely on concentration gradients to bias their motion towards food sources or away from repellents. Various models such as run and tumble, run and reverse, and run and flick, have been proposed to describe bacteria's motion in a local concentration gradient. Our first model proposes a two-state run and tumble model to describe a phenomenon called the Volcano Effect, wherein bacteria overshoot a food source and exhibit erratic behavior within a certain radius range centered at the food source. Our second model seeks to show that banding occurs in bacteria with chemotactic receptors that reach saturation near the source. We expect to see an annulus of bacteria around the food source, gathering densely within a certain radius range from the food source. Early results of the run and tumble portion of our model have produced biased random walks and distributions of bacteria populations near high chemical concentrations. Our models aim to accurately include biological phenomenon such as saturation, advection, and moving point sources. Our next goal is to model the run and flick movement.

*Research Institution: IUPUI
Research Advisor: Steve Pressé

The delta and the tripod are extremal
Baptiste Dejean*, *Indiana University, Bloomington*

When is one rubber band network "looser" than another in the sense that it always has less potential energy than the other? When networks are allowed to stretch on a line, they behave like resistor networks, which are well-understood. We used this case and Euclidean geometry to relate rubber band networks in the more complicated case that they are stretched in a combinatorial tree. We found that for any network, if three points are fixed, the network's energy is sandwiched between the energies of the electrically equivalent tripod- and delta-shaped graphs.

*Research Institution: Indiana University
Research Advisor: Dylan Thurston

A novel description of knotted non-orientable surfaces in S^4
James Dix*, *University of Texas, Austin*

This presentation describes the construction of spun Klein bottles generated from an inversion of a knot in S^3. Several perspectives of the same construction are described, using banded link diagrams or surgery on S^3 x S^1. This perspective can be used to describe the spun projective planes constructed by Price and Roseman, and explain the different surfaces generated by a spun trefoil. This construction gives several potentially interesting knotted Klein bottles which may not decompose into a connected sum of projective planes. At the very least, no obvious decomposition has been found for several surfaces.

*Research Institution: Indiana University
Research Advisor: Jeffrey Meier

Using a Theoretical Model to Predict Transplant Rejection
Olga Dorabiala*, *Pennsylvania State University*

Organ transplantation is a life-saving procedure that restores the functionality of an organ. However, transplant patients must remain on immunosuppressive drugs for their lifetime, causing them to be at an increased risk for infections and other health complications. Alternative treatment strategies that promote transplant acceptance without compromising the patient's immune system are therefore a great medical need. Here, strategies for the adoptive transfer of regulatory T cells are theorized using an experimentally-based model of the immune response to murine heart transplants. The model predicts the success of different dosing strategies of regulatory T cells on graft survival. Timing, dosage, type, and location of regulatory T cells are all factors that are assessed using the model. The model predicts that adoptive transfer of activated regulatory T cells administered directly to the graft is most effective at extending graft survival. Furthermore, the model shows a non-monotonic relationship between graft survival time and the time at which activated regulatory T cells are administered to the graft. Finally, varying the duration over which a dose of regulatory T cells is administered to the graft has a minimal effect on the length of graft survival, suggesting that the dose amount is more influential than timing.

*Research Institution: IUPUI
Research Advisor: Julia Arciero

Intrinsic Surfaces of Revolution
Daniel Freese*, *Liberty University*

We generalize the Enneper surface by studying intrinsic surfaces of revolution whose principal curvatures are independent of rotation and whose principal curvature directions depend only on rotation. We show that, in order to satisfy the Gauss and Codazzi equations, such surfaces have constant mean curvature and rotate at constant speed, except in specific cases. We can construct and classify the minimal surfaces using Bonnet's Theorem. We cannot find explicit equations for surfaces in the general constant mean curvature case, but we can construct them numerically and classify them as Smyth surfaces. We finally consider the special case where principal curvature directions do not rotate and find a general surface of revolution isometric to Enneper's surface.

*Research Institution: Indiana University
Research Advisor: Matthias Weber

A Database of Toroidal Belyi Pairs
Ivan Gonzalez*, *Florida International University*
Dionel Jaime*, *University of Rochester*
Gabriel Ngwe*, *Williams College*

A Belyi map $\beta: P^1(C) \to P^1(C)$ is a rational function with at most three critical values; we may assume these values are $\{0, 1, \infty\}$. Replacing P^1 with an elliptic curve E: $y^2 = x^3 + A\,x + B$, there is a similar definition of a Belyi map $\beta: E(C) \to P^1(C)$. Since $E(C) \cong T^{\wedge 2}(R)$ is a torus, we call (E, β) a toroidal Belyi pair. This project seeks to create a database of such pairs.

For each positive integer N, there are only finitely many toroidal Belyi pairs (E, β) with $\deg \beta = N$. Using the Hurwitz Genus formula, we can begin this database by considering all possible degree sequences on the ramification indices as multisets on three partitions of N. For each degree sequence, we compute all possible monodromy groups $G = \mathrm{im}[\pi_1(P^1(C) - \{0, 1, \infty\}) \to S_N]$. Finally, for each possible monodromy group, we compute explicit formulas for Belyi maps $\beta: E(C) \to P^1(C)$ associated to some elliptic curve E: $y^2 = x^3 + A\,x + B$.

*Research Institution: Purdue University
Research Advisor: Edray Goins and Mark Pengitore

Elastic Networks and Train Track Structures
Christian Gorski*, *Notre Dame University*

We define a notion of Dirichlet energy and harmonic maps for maps from marked graphs with an “elastic structure” to marked metric graphs. In the case that the target graph is simply an interval with some marked points, there is a direct analogy to the study of resistor networks. In this case, the energy of the harmonic map is a quadratic function of the relative positions of the marked points on the interval. More generally, the energy of the harmonic representative of a given homotopy class is a piecewise quadratic function of the lengths of edges of the target graph. This function can be computed by examining ``train track structures" on the source graph.

*Research Institution: Indiana University
Research Advisor: Dylan Thurston

Voting Power in Norway’s Parliament
Jon Kaasa*, *Goshen College*

The idea of Democracy is interchangeable with equal or “fair” representation. What does equal representation look like in a political system with multiple parties, millions of votes and only a limited integer number of seats to be allocated? We investigate the current apportionment method used in Norway: a proportional representation system where the unit of representation is bound both to territory and to party. The voting power of a political party is not necessarily proportional to the number of seats it controls, and so we make use of power indices defined by Shapley-Shubik, Banzhaf, and Deegan-Packel. In the given context of Norway, we investigate how close the voting powers of the political parties correspond with the number of votes they received, and when it is advantageous for parties to merge together to increase combined voting power rather than stay separated.

*Research Institution: Goshen College
Research Advisor: David Housman

Clairaut's Investigation of Tangents and Properties of Various Curves
Taner Kiral*, *Wabash College*
Jonathan Murdock*, *Wabash College*

In 1726, Alexis Clairaut presented four new curves to the Royal Academy of Sciences in Paris, at the age of twelve and a half. He subsequently published article featuring these curves in 1734. Clairaut gives the algebraic and geometric properties of each curve, and shows how they can be used to solve the ancient problem of finding any number of mean proportionals between two given straight lines. He also finds tangents to his curves, and computes their quadratures (integrals).

In our presentation, we will briefly provide background, and then delve into the first of the four problems that Clairaut presents and the derivations he provides. We will then relate the next three problems back to the first and explain the conclusions Clairaut arrived at for each of the problems.

*Research Institution: Wabash College
Research Advisor: Colin McKinney

Network Classification and Inferencing
AnnaLee Knapp*, *Valparaiso University*
Sammantha Nowak-Wolff*, *Valparaiso University*

Currently, there is no definitive method for classifying networks into distinct categories. The leading method in network classification involves using Support Vector Machines (SVM) to identify subgroups within a broader category, often a specific field of investigation. By looking at data mining classification methods, and feature analysis, this work seeks to classify networks into meta-categories with high accuracy.

Preliminary investigations have been conducted with the Network Repository data from BHOSLIB, DIMACS, DIMACS10, Retweet Networks, Social Networks, and Temporal Reachability networks. SVM has revealed promising results with a classification accuracy of 67.5%. This was achieved by excluding Total Triangles. In addition, Naive Bayes has shown good results with the exclusion of the attribute maximum triangles. It has produced a classification accuracy of 84.3%.

Identifying the best algorithm and the best features to consider will lead to a more procedural and efficient way of classifying graphs into these meta-categories. This will be useful to the wider scientific community by allowing them to more easily choose effective algorithms for graph mining and investigations.

*Research Institution: Valparaiso University
Research Advisor: Karl Schmitt

Monodromy Groups of Toroidal Graphs

Caitlin Lienkaemper*, *Harvey Mudd College*
Baiming Qiao*, *Purdue University*

A graph is said to be planar if it can be embedded in the plane, or equivalently, on the sphere such that its edges do not cross. In the same manner, we can define toroidal graphs as those graphs which can be embedded on the torus without edge crossings. Bipartite graphs embedded in Riemann surfaces, such as the sphere or the torus, can be viewed as Dessins d'Enfants, graphs that encode the ramification structure of rational maps known as Belyi maps.

In this talk, we discuss three infinite families of regular toroidal Dessins. Given a toroidal Dessin with $N = |E|$ edges, we can label the edges E to find a particular group $G = \text{im}[\pi_1(P^1(C) - \{0, 1, \infty\}) \rightarrow S_N]$ called the monodromy group; it is the ``Galois closure" of the group of automorphisms of the graph. We will discuss some of the challenges of determining the structure of these groups, and present visualizations of group actions on the torus.

*Research Institution: Purdue University
Research Advisor: Edray Goins and Mark Pengitore

Inverse Problems in Electrostatics

Megan Masterson*, *Case Western Reserve University*
Theodore Rogozinski*, *Case Western Reserve University*

This study focused on the creation and classification of degenerate equilibria curves of the electrostatic force from point charges. New zero sets were discovered using algebraic manipulation and *Mathematica* programs that plotted the intersections of the zeros of the forces in different directions. Exploration of these zero sets revealed the existence of infinite equilibrium lines, single circles of equilibria, and multiple circles. Depending on the configuration, either planar or linear, the circles were contained in the same plane or in planes parallel to one another. Determining which configuration results in which zero set is typically dependent on the symmetry of the configuration. These various zero sets that were formed by different configurations give insight into the necessary sampling that must be done in order to determine the unique configuration that created the field.

*Research Institution: IUPUI
Research Advisor: Joseph Rosenblatt

GTR+Γ+I Model Identifiability using a Discrete Gamma Distribution
Kees McGahan*, *Willamette University*

Phylogenetics is the study of trees that describe the evolution of organisms. Mathematics provides models for DNA evolution that enable Biologists to determine trees in a rigorous statistical framework.
In order for these models to be useful, the models need to be identifiable. That is, with an infinite amount of data from the true model, we should be able to recover all the parameters in that model uniquely. One commonly used model, especially in phylogenetic software, is the GTR+Γ+I model, whose parameters include: T, the phylogenetic tree, Q, the General Time Reversible (GTR) Markov rate matrix, and R, a random mutation rate taken from a mixture of a Gamma distribution and invariant sites. This model has already been shown to be identifiable when the Gamma distribution is continuous. However, all known software packages use a discrete version of the Gamma distribution instead. In this REU project, we are exploring both numerically and analytically whether the model with only two rate classes and no invariant sites is identifiable.

*Research Institution: Indiana University
Research Advisor: Elizabeth Housworth

Component Analysis of Local Field Potentials in Prefrontal Cortex and Ventral Tegmental Area
Matthew Moss*, *Stony Brook University*

Cellular processes in neural tissues generate movement of electrical charges. Multiple techniques for measuring this electrical activity allow for characterization of cellular processes at different scales of the brain tissue. In particular, electrodes inserted deep into the brain record electric fields referred to as the local field potential (LFP; Buzsáki, et al., 2012). It is believed that LFP characterize inputs to the network of neurons surrounding the electrode. However, the contributions of other sources to LFP are actively discussed. In cortical networks, LFPs are strongly correlated with another measure of the electrical activity spike recordings from individual neurons. Thus, the neurons contribute to LFP, and it is possible to figure out their individual contributions through the use of a linear filtering mechanism (Rasch, et al., 2009). First, we successfully repeated this analysis in the rat Prefrontal Cortex (PFC). However, this has proven not to be possible in other areas, such as the midbrain. In particular, neurons in the Ventral Tegmental Area (VTA) are not as well spatially aligned as cortical neurons and fire mostly asynchronously, which leads us to believe that their electric fields may cancel instead of summing in the LFP readings. Because of this, the local LFP signal may be dominated by volume conducted signals from other brain regions. A combination of current source density (CSD) analysis and Independent Component Analysis (ICA) has been used in previous work by Łeski et al. (2009) to solve this problem. We used this approach on the VTA single unit and LFP signals. This method allowed us to find the specific sources of LFP signals in the VTA and the contribution of particular neurons to the cumulative LFP.

*Research Institution: IUPUI
Research Advisor: Alexey Kuznetsov

Fluid flows over an osteocyte: modeling and simulation
Maddie Sanden*, *University of Kentucky*

An osteocyte, living in a fluid-filled matrix composed of the lacuna and many radiating canaliculi, has been found to be responsible for mechanical sensing leading to bone growth or decay. However, it is not well understood which part of the osteocyte actually senses the mechanical forces. Due to the complexity of the fluid-osteocyte-lacuno-canalicular (FOLC) system, it is difficult to study the FOLC system in situ; instead, mathematical modeling and computer simulation is viable. In this talk, we discuss a FOLC model in 2D and interactive simulation by the lattice Boltzmann (LB) method of fluid flow over the osteocyte in the Lacuna-canalicular system. Velocity/vorticity fields, in particular, the wall shear and normal forces applied by the flow on the osteocyte surface, are all computed and visualized. The influences of inlet/outlet boundary conditions, number and geometry of the canaliculi are investigated.

*Research Institution: IUPUI
Research Advisor: Luoding Zhu

Two-compartment model of dopamine release: dependence on firing pattern
Angela Wei*, *University of Kentucky*

Dopamine (DA) throughout the brain is provided by dopaminergic neurons located in two well-defined nuclei of the midbrain. Dopaminergic neurons release DA both at their synaptic terminals and extrasynaptically. These neurons fire in two distinct patterns: tonic and burst (Grace and Bunney 1984a,b). Tonic firing determines background levels of DA; burst firing determines transient DA increases (Schultz 2002). When the firing of dopaminergic neurons is manipulated to be more bursty, DA levels measured by microdialysis are unaffected (Floresco et al. 2003). However, when burstiness is increased to the same levels while DA reuptake is blocked, microdialysis measurements show that the DA levels increase three-fold. Microdialysis probes can only measure extracellular DA at very low temporal and spatial resolution; these limitations then prevent microdialysis probes from providing information about the mechanism behind the effect of uptake inhibitors. The purpose of this project is to investigate this mechanism behind the effect of DA uptake inhibitors through computational modeling. A two-compartment model is used to show the effects of manipulating firing from tonic to bursting on average synaptic and extrasynaptic DA levels. Two cases were studied both analytically and in direct simulations. The first case assumed that synaptic DA release is negligible because of how small the synaptic volume is. The second case assumed extrasynaptic DA release to be negligible while synaptic DA release was high in order to reveal the contribution of DA diffusing out of the synapses. Both analytical and computational methods were coded in MatLab and their solutions were then compared. From the first case, levels of average extrasynaptic DA were proven to be independent of the firing pattern for any physiologically-relevant parameters for both the analytical and computational models. In the second case, we found that an increase in burstiness was followed by an increase in average synaptic DA levels. The model reveals that DA is released in the synapses and spreads by diffusion, but is not directly released into the extrasynaptic space. This movement of synaptic DA creates the dependence on the firing pattern observed in experiments.

*Research Institution: IUPUI
Research Advisor: Alexey Kuznetsov