ABSTRACTS

(*in alphabetical order*)

1. Properties of Laplacian simplices formed from joined-path graphs

Samikshaya Ananthakrishnan[*], *University of California, Berkeley*

A polytope is a geometric object defined as the convex hull of a set of vertices. Simplices are polytopes with affinely independent vertices that span the dimension the polytope is in. The simplex of interest in this research is the Laplacian simplex. Laplacian simplices are formed from graphs by constructing a Laplacian matrix and then deleting one row; Laplacian simplices constructed from joined-path graphs have patterns in relation to reflexivity. These patterns can be understood based on the hyperplanes of these simplices, which I generated from the software Sage. Furthermore, based on the hyperplane descriptions it can be discovered when the simplices are reflexive as the number of paths in the respective joined-path graphs varies.

[*]Research Institution: Rose-Hulman Institute of Technology
Research Advisor: Dr. McCabe Olsen

2. Properties relating to chip-firing on tree graphs

Elijah Atkins[*], *Greenville University*

In this presentation I will discuss some background in graph theory before introducing a theorem about the number of reachable chip configurations for a specific type of tree graphs. I will also introduce some information about Ehrhart theory, including polytopes, lattice point enumerators, and the Ehrhart series. I will close with some conjectures about the properties of the h* polynomial in the Ehrhart series associated with the polytope that corresponds to the reachable chip configurations of a tree graph.

[*]Research Institution: Rose-Hulman Institute of Technology
Research Advisor: Dr. McCabe Olsen

3. Using interpolation to combine macro and microscale models for osteocytes

Sophie Boileau[*], *Carleton College*
Elizabeth Tootchen[*], *Saint Joseph's University*

Osteocytes are cells nested deep within bone. When external mechanical forces are applied (e.g., walking), osteocytes respond (mechanotransduction) by releasing signals that initiate bone remodeling. The process by which macroscale forces are amplified and localized in order to induce osteocyte mechanotransduction, however, is not yet fully understood. To investigate this, our group is creating a three-dimensional multiscale osteocyte model. The microscale component models the osteocyte and the thin layer of interstitial fluid that surrounds it. The macroscale component models

the poroelastic (sponge-like) bone matrix that encases the osteocyte and surrounding interstitial fluid. To couple the two models, the displacements and velocities on the wall bounding the interstitial fluid (lacuna-canalicular wall) in the macroscale model are used in the microscale model, while the force estimates along that same wall from the microscale model are used in the macroscale model. In this presentation, we discuss our work on developing tools that can be used to transfer quantities between the macroscale and microscale models. We have used Matlab to create and use linear interpolation tools to transfer displacements and velocities from macroscale to microscale meshes and forces from microscale to macroscale meshes. In addition, to improve force smoothness, we also consider using radial basis functions to estimate forces and derivatives on the microscale mesh before transferring them to the macroscale mesh. Coupling these tools with the two models will help give insight into therapies for patients with sedentary lifestyles and bone-related diseases, like osteoporosis and leukemia.

[*]Research Institution: IUPUI
Research Advisor: Dr. Jared Barber

4. Improving forecasts for the 2022 midterm elections

Ryan Branstetter[*], *Purdue University*
Mengqi Liu*, *Purdue University*
Manas Paranjape*, *Purdue University*

Forecasting the results of United States elections is an interesting problem that draws on many fields, including mathematics. An existing mathematical approach (by Volkening *et al.*) based on an adapted disease-transmission model has comparable accuracy to popular forecasters. This model, a system of differential equations, depends on polling data to specify parameter values. Currently, the model treats every poll as equally accurate, but some pollsters are more reliable than others or have dependable partisan leans. This motivates our project: we aim to adjust the polling data to produce better forecasts and forecast the 2022 midterms. To do this, we calculate the average bias of each pollster across their polls from 2004–2020. We implement three different methods of computing biases to generate forecasts of past elections. For our analysis of the 2020 Senate and governor elections, we find that accounting for pollster bias improves the original method, in which polling data is not adjusted at all. Our results suggest that adjusting the polling data by past pollster bias may be an effective method to improve accuracy. Using our new method, we plan to post forecasts regularly in the months leading up to the election on November 8. To this end, we are building a responsive webpage using Bootstrap to serve as the home for our forecasts. By combining JavaScript and CSS, we are incorporating interactive elements into our website, including "Election Roulette". Our animations help demonstrate that forecasts are not certain, but probabilistic.

[*]Research Institution: Purdue University
Research Advisor: Dr. Alexandria Volkening

5. Dopamine modulation of basolateral amygdala activity *in vivo* and *in vitro*

Daniel Brin*, *University of Chicago*
Emily Pecsok*, *Cornell University*

Dopamine (DA) is a neurochemical substrate for motivation and saliency, but understanding of the mechanisms of action of DA is far from complete. DA release in basolateral amygdala (BLA) is associated with increased alcohol consumption. Now, there is a discrepancy between in vitro and in vivo studies on the neurophysiological effects of DA on neural firing rate. It is well established in rats that DA increases the firing rate of principal neurons in vitro but decreases it in vivo. To unify previous studies, we model neural activity with the firing rate formalism and use phase plane analysis to visualize steady states in baseline conditions as well as during DA application and blockade of inhibition, which mimics application of bicuculline in vitro. We separate the BLA into the mutually inhibiting populations of fear neurons (BAf) and extinction neurons (BAe). In the transition from in vivo to in vitro, the emergence of the new equilibria occurs via a pitchfork bifurcation. The BLA population splits into active and passive groups, and this works as a mechanism that allows the active group to escape inhibition and increase its firing during DA application. The conditions in the in vivo study may not be behaviorally relevant since the rats were treated with anesthesia. Now, in behaviorally relevant conditions where BLA receives strong inputs, DA strengthens amygdala response and enhances corresponding behavioral choices, in accord with in vitro experimental results.

*Research Institution: IUPUI
Research Advisor: Dr. Alexey Kuznetsov

6. Point estimators for dynamical entropy

Patrick Conway*, *University of Notre Dame*

The entropy of a dynamical system is a quantitative measure of the degree of chaotic behavior and uncertainty in the trajectory of the system. There has been a rich history of developing point estimators for the entropy of fixed distributions. In this talk, I will explore how such point estimators could be applied to estimating the entropy of dynamical systems. I will also show that given a suitable estimator and a sufficiently large sample size, there exists a large subsample satisfying the Asymptotic Equipartition property for which the estimated entropy of the subsample is close to the actual entropy of the system.

*Research Institution: Indiana University Bloomington
Research Advisor: Dr. Salman Siddiqi

7. Invariant expanding graphs in Julia sets

Helen Dai*, *Harvard University*

For any complex-valued function, its Julia set is the set of "unstable" points under iteration. The Julia set of a large class of these functions, called hyperbolic, can be expressed as a series of graph approximations, and hyperbolic maps are notable for having a definite expansion acting on their Julia set. We quantify this expansion using these graph approximations and, in particularly nice cases, find an invariant graph in the Julia set exhibiting this expansion.

*Research Institution: Indiana University Bloomington
Research Advisor: Dr. Dylan Thurston

8. Modeling treatment strategies for transplant patients

Danielle DaSilva*, *Elon University*
Lia Rotti*, *Tufts University*

The standard of care for solid organ transplants is life-long immunosuppression to prevent acute graft rejection. However, immunosuppression increases the risk of complications from opportunistic infections and other diseases. Thus, finding alternative therapies to decrease dependence on immunosuppression is a desirable clinical goal. One possible alternative is the adoptive transfer (AT) of regulatory T-cells (Tregs), through which a high dose of Tregs is administered to protect the graft. However, this strategy has been largely unsuccessful when used as a monotherapy. Our research project uses mathematical modeling to reproduce the clinical outcome of graft survival with immunosuppression and to understand the mechanisms of the immunosuppressive drug Cyclosporine A (CsA) in a murine heart transplant. We also model the use of AT in combination with immunosuppression to predict combination therapies that promote graft acceptance while minimizing the use of immunosuppression. The impact of different immunosuppression dosing strategies, including constant, tapered, bolus intravenous, and bolus oral dosing, is simulated in the presence and absence of AT. Immunosuppression and AT factors (such as dose level, mechanism of action, timing, and frequency) are varied. The model was used to predict the immunosuppression dosing levels at which AT could prevent graft destruction for different immunosuppression administration types. This work provides a framework for better understanding the dynamics and mechanisms of immunosuppressive drugs and AT in transplantation. This theoretical model is a first step in predicting treatment using AT in combination with immunosuppression and can guide future experimental design for therapeutic interventions.

*Research Institution: IUPUI
Research Advisor: Dr. Julia Arciero

9. Dynamic phase transitions in a parabolic PDE system

Adam Earnst*, *University of Michigan*

Using dynamic phase transition theory, I explore and classify the phase transitions between stable states in a dissipative parabolic partial differential equation when the control parameter crosses a certain critical threshold. In particular, I first develop a method to compute a leading-order approximation of a center manifold to which we can reduce any PDE of interest. This reduced

equation can then be analyzed at the critical value of the control parameter to classify a phase transition in one of three categories: continuous, jump, or mixed. Using the blow-up method to investigate degenerate fixed points, we can find that for most tuplets of coefficients, the parabolic system undergoes a jump or mixed transition. In addition to these analytical methods, I use Mathematica and Matlab to visualize the behavior of the reduced ODE through phase portraits and numerical approximation.

*Research Institution: Indiana University Bloomington
Research Advisor: Dr. Shouhong Wang

10. Effects of the Malawi Social Cash Transfer Program on food prices
Lillian Gibson*, *Grinnell College*

The Malawi Social Cash Transfer Program (SCTP) is a nation-wide anti-poverty initiative consisting of welfare payments made directly to the poorest 10 percent of the country's population. The program has been proven to make a substantial impact on the depth of poverty and hunger experienced by families receiving the transfers. One potential downside of this program is the potential to cause inflation by increasing the demand for goods in the area. Rising prices could undo the increased purchasing power of recipients and negatively impact ineligible members of the community. However, some recipients invest the money in small farms and businesses, providing a concurrent positive supply shock. This obscures the true effect of the cash transfers on the larger economy. Through an event study and further data analysis on inflation, I investigate the effects of the program on food prices in Malawi.

*Research Institution: Rose-Hulman Institute of Technology
Research Advisor: Dr. Wayne Tarrant

11. Making simulated patterns look real: image processing for zebrafish
Caroline Henson*, *Purdue University*

Zebrafish (*Danio rerio*), named after the striped patterns that form along their body, are small fish with important biomedical applications. Zebrafish patterns are comprised of pigment cells. One type of pigment cell, melanophores, is closely related to skin cancer cells in humans. Due to the small size of these fish (less than 3cm in length), understanding how cells interact and develop to form patterns is a challenging endeavor. While there are mathematical models that describe pattern development, images of these simulations do not achieve the variety in cell size or color seen in real fish. The goal of my research is to develop post-processing software to make simulated zebrafish patterns look akin to live fish. By viewing a large number of zebrafish images, I am analyzing their cell colors and shapes. Based on the mathematical model data, I build realistic simulated fish patterns by reconstructing small portions of cells in real zebrafish. Additional filters are then overlaid onto the simulated images to replicate the glossiness and form of live fish. Our software produces more convincing images of zebrafish and has potential applications in future zebrafish study.

*Research Institution: Purdue University
Research Advisor: Dr. Alexandria Volkening

12. Desynchronization patterns of the brain

Katrina Hoetmer*, *IUPUI*
Kelly Ryan*, *Butler University*

Neural synchrony is believed to be an important mechanism underlying many phenomena in the human brain. Neural signals of the brain go in and out of synchrony due to a variety of factors. Synchrony is a non-instantaneous phenomenon and from the data analysis perspective, one considers synchrony in a statistical sense, observed over a sufficiently large number of cycles of oscillations. Yet if this synchrony is present on the average, one can look at each cycle of oscillations and see how far away it is from a synchronized state. Transitions between synchronization and desynchronization intervals (and, therefore, temporal patterns of the phase locking) can be represented using the Markov chain model and associated transition matrix. The magnitude of the second largest eigenvalue of this matrix defines the speed of convergence of the perturbed system to the equilibrium state. This eigenvalue was analyzed in dependence on the transition rates, in particular, the case of a large sample of randomly generated transition rates similar to those observed experimentally. The analysis was primarily performed in Matlab. Earlier studies suggested that the absolute value of the second largest eigenvalue can be very small (pointing to potentially very fast convergence of the perturbed functional brain networks to the equilibrium distribution). This study shows that, based on the previous experimental reports of the transition rates, this eigenvalue may not necessarily have a very small magnitude. More precise comparison with experimentally obtained transition rates may help to better understand the convergence of the perturbed brain networks to a rest state. However, the slightly larger convergence rates may point to the potential trade off in the convergence speed as the brain optimizes the information processing in the presence of multiple constraints.

*Research Institution: IUPUI
Research Advisor: Dr. Leonid Rubchinsky

13. Covariance of coalescence times of loci in the Wright-Fisher Model

David Kogan*, *University of Pennsylvania*

Tajima's estimator for the population-scaled mutation rate θ is the average the number of pairwise differences in nucleotides at n unlinked loci (fixed location in the chromosome). The best way to increase the accuracy of the estimator is to increase the number of pairs of unlinked loci hence taking $n \to \infty$. It turns out (King et al. 2018) that the variance of Tajima's estimator ($\hat{\theta}$) is nonzero even as $n \to \infty$. This is a result of the small correlation between coalescence times at the unlinked loci. We consider the diploid, discrete-time, Wright Fisher Model with one sex, which allows for recombination and selfing. We let T_i to be the time until coalescence at locus i in generations under this model. Assuming the loci are unlinked we compute the values of $\mathrm{Cov}[T_i, T_j]$ and $\mathrm{Corr}[T_i, T_j]$ in terms of the selfing probability σ_N and population size N. This strengthens the results found by King et al. (2018) and gives explicit values for $\mathrm{Var}[\hat{\theta}]$.

*Research Institution: Indiana University Bloomington
Research Advisor: Dr. Wai-Tong (Louis) Fan

14. Self-reachable chip firing configurations on trees
Benjamin Lyons*, *Rose-Hulman Institute of Technology*

We investigate self-reachable chip-firing configurations, which are chip configurations that can revert to themselves after a nonzero number of firings, on finite trees. Our main goal is to analyze the convex hull of all self-reachable configurations with a particular number of chips on a particular tree. We prove that all of the lattice points in this convex hull are self-reachable configurations. Furthermore, we conjecture that this polytope has the integer decomposition property. We also conjecture that for a fixed number of chips, across all trees with the same number of vertices, these convex hulls share the same Ehrhart series.

*Research Institution: Rose-Hulman Institute of Technology
Research Advisor: Dr. McCabe Olsen

15. A time-dependent three-dimensional model for linearly elastic shells
Aaron Meixner*, *The Ohio State University*

Our work focuses on the contrivance of a suitable numerical scheme for predicting the deformation of a time-dependent problem governing the displacement of a three-dimensional linearly elastic flexural shell. First, the rigorous variational formulation of the time-dependent problem for the three-dimensional cylindrical shell under examination is stated. Second, the existence and uniqueness of the solution of one such problem is recalled. Third, a full discretization with respect to the space variable and the time variable for approximating the evolutionary process under consideration is given, and numerical simulations are presented.

*Research Institution: Indiana University Bloomington
Research Advisor: Dr. Paolo Piersanti

16. Osteocyte process modeling
Maxim Mukhin*, *Vanderbilt University*

Osteocytes are bone cells that reside in fully mature bone tissue. They are located inside the bone matrix in small pockets called the lacuna, with cytoplasmic processes located inside channels called the canaliculi. Osteocytes can sense stress and strain applied by the interstitial fluid flow, leading to bone remodeling. However, the stress and strain at tissue level have to be amplified by ten times for the osteocyte to have response in vivo. The stress/strain amplification mechanism is not well understood yet. The osteocyte has many seemingly randomly oriented processes and each resembles a tapered porous cylindrical structure submerged in fluid, surrounded by a rough canalicular wall. This project focuses on 3D modeling and simulation of a cellular process interacting with a fluid flow in a canaliculus. The process is modeled as a fibrous gradually tapered cylinder. The process can be arbitrarily oriented with respect to the cell main body. The roughness of the canalicular wall is modeled by randomly generated wall thickness. The fluid is modeled as a viscous incompressible Newtonian fluid. The flow is modeled by the lattice Boltzmann approach (D3Q19 model). The fluid-structure-interaction is handled by the immersed boundary (IB) method. Our preliminary results show a significant increase in deformation of the cellular process when the canalicular wall is rough, compared to the smooth case. This allows us to better understand stress/strain amplification

mechanisms. This model is part of a more comprehensive multiscale integrative model of osteocyte-fluid-interaction in a lacuno-canalicular network under cyclic loading.

*Research Institution: IUPUI
Research Advisor: Dr. Luoding Zhu

17. Guts poker and generalized recursive games
Jacob Platnick*, *Northwestern University*
Jay Hwan Lee*, *University of California, Berkeley*

Game Theory has been a subject of study and discussion since the work of Borel and von Neumann. One type of game that has been well studied is the recursive game, where summing the payoff over repeated rounds of play gives the final "value" of the game. However, this analysis requires the assumption that the stakes of the game decrease with each round, essentially guaranteeing a stopping point. Motivated by Guts poker, a game of increasing stakes, we aim to study a generalized Recursive Game without this assumption.

*Research Institution: Indiana University Bloomington
Research Advisor: Dr. Kevin Zumbrun

18. Conditional coalescent theory in a population of varying size
Daniel Rickert*, *Indiana University Bloomington*

Imagine taking a sample of genes from a certain population. If we trace these genes back in time through past generations up to their most recent common ancestor, what does the resulting tree look like? Coalescent theory is the mathematical study of questions of this type. The bulk of coalescent theory has dealt with constant population size models that give rise to the Kingman coalescent in the large population limit. However, real populations (e.g., the human population) rarely have constant size. Furthermore, for practical applications, the sample types are often already known (e.g., the blood types for a sample of individuals), so it is useful to understand the coalescent process between individuals in the sample conditional on such information. As many nice properties of the Kingman coalescent are lost in this generalization, our research focuses on small and limiting cases to make larger conjectures about the behavior for arbitrary sample sizes and arbitrary population sizes.

*Research Institution: Indiana University Bloomington
Research Advisor: Dr. Wai-Tong (Louis) Fan

19. Diameter sensitivity in dual-wavelength retinal oximetry
Benjamin Shoemaker*, *IUPUI*

Dual-wavelength retinal oximetry is a technique used to produce oxygen saturation maps of the retinal vasculature. In small diameter vessels (<100 μm), an artificial increase in these oxygen saturation measurements has been observed; this will be referred to as the diameter artifact. To characterize this artifact, the following data was collected from ten healthy patients using Oxymap Analyzer: percent oxygen saturation, diameter, position, and optical density ratio (ODR, a ratio between the returned

intensity of light at two wavelengths) at multiple points along the retinal arteries and veins. The percent oxygen saturation is derived directly from the measured ODR. In some cases, negative values of ODR are returned by the oximetry software, which can lead to instances of non-physiological values for oxygen saturation (>100%). This study showed the impact of removing these negative values on the diameter artifact. Additionally, a conceptual model considering the major light paths (e.g., single pass, double pass, and backscatter) taken by a photon in and around the retinal tissue is developed to identify additional factors leading to a diameter artifact. Reflectance coefficients are model parameters that refer to the impact of each light path on the total returned light intensity. The impact of varying these parameters is demonstrated, and optimum values are obtained by minimizing the squared error between model and experimental data. Ultimately, this study aids the understanding of retinal tissue optics and the diameter artifact, providing a first step toward improving the utility of retinal oximetry in the diagnosis of ocular pathologies.

[*]Research Institution: IUPUI
Research Advisor: Dr. Julia Arciero

20. Constraints on n-lattice simplices of volume 1/n! in n-cube, unique under symmetry
David Snider[*], *University of North Carolina at Chapel Hill*

For each n, the number of n-simplices of volume $1/n!$ with vertices in $[0,1]^n \cap \mathbb{Z}^n$ unique under symmetry is unknown. This project introduces a multigraph representation of such simplices and a few results on this representation, including formulas for its characteristic polynomial. A conjecture is stated that would put a tighter bound on the number for arbitrary n.

[*]Research Institution: Rose-Hulman Institute of Technology
Research Advisor: Dr. McCabe Olsen

21. Simplices from circulant matrices
Robin Steuteville[*], *Northwestern University*

In this talk we will consider polytopes formed from circulant matrices made from the vector ($v - 1$, $v-2$, ..., 1, 0). The polytope we will consider is the convex hull of all the columns of these circulant matrices. We will see that these polytopes are simplices and are reflexive in the case where v is odd. We will further conjecture a form for the Ehrhart series of the polytope and that it has other properties of interest, such as the integer decomposition property.

[*]Research Institution: Rose-Hulman Institute of Technology
Research Advisor: Dr. McCabe Olsen

22. Microfinance, GDP, and informal economies
Joseph Tarrant[*], *Rose-Hulman Institute of Technology*

Since its innovation around 50 years ago, Microfinance has been used throughout the world as a potential method of helping those living in poverty better their situation in life. Especially in the last 20 years, Microfinance has seen a drastic increase in popularity as people throughout the world search

for ways to bring about an end to poverty. However, despite its increased use, there has been little research done regarding whether Microfinance is beneficial to those living in poverty or to the economies of the nations where it is most heavily utilized. We attempted to determine what, if any, benefits Microfinance brought to economies and to those living in poverty by looking at potential correlations between the aggregate Microfinance notional of a country and the GDP of said country as well as the size of its informal economy.

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23. Testing hyperinflation data against Makochekanwa's Model

Laura Vaughan*, *Vanderbilt University*

Hyperinflation has become an increasingly common problem in the last century. In 2007, Zimbabwe entered a period of extreme hyperinflation that led to the collapse of the Zimbabwe dollar. In this research, I evaluate Makochekanwa's (2007) model that formed hypotheses surrounding Zimbabwean inflation from 1999-2006. I found updated data and work to find good estimators for missing datasets. I measure the model's goodness of fit to the new data and find that one variable has vast significance over all others. I then test other hyperinflationary episodes with similar available data to determine if Zimbabwe was an outlier case. Continuation of this research involves testing further hyperinflationary episodes and developing a model that better encapsulates extreme hyperinflation.

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24. The impact of asset liquidity on healthcare-seeking behavior in rural Nigeria

Grace Wolfe*, *University of South Florida*

In rural Nigeria, both informal and formal healthcare are available to the population, but informal healthcare is far more utilized. Literature suggests that a major barrier to formal healthcare access for those in poverty is cost. This study is a health econometric data analysis of the Financial and Health Diaries 2012-2013 conducted in rural Nigeria, seeking to understand how asset liquidity influences an individual's decision to seek formal over informal providers.

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